

Name:

Date:

Period: 6

Properties of Parallelograms

Learning Objective INBAT use properties of parallelograms to find side lengths and angle measures; use parallelograms in the coordinate plane.

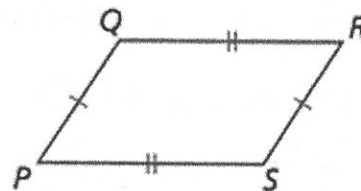
Parallelogram: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

DEFINITION

The theorems below describe other properties of parallelograms.

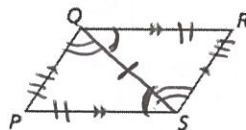
Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then the opposite sides are congruent.



PROOF: Given: PQRS is a parallelogram

Prove: $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$



Statements

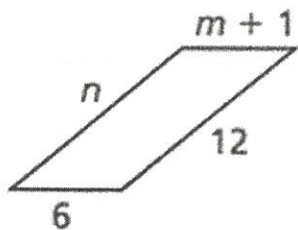
Reasons

- ① PQRS is a ||-gram
- ② Draw $\overline{QS}$
- ③ $\angle RQS \cong \angle PSQ$ ($\overline{RQ} \parallel \overline{PS}$)
 $\angle RSQ \cong \angle PQS$ ($\overline{PQ} \parallel \overline{RS}$)
- ④ $\overline{SQ} \cong \overline{QS}$
- ⑤ $\triangle PSQ \cong \triangle RQS$
- ⑥ $\overline{RQ} \cong \overline{PS}$ and $\overline{PQ} \cong \overline{RS}$

- ① given
- ② Thru any 2 points there is one line
- ③ if $\parallel \rightarrow$ alt. int. $\angle$ s $\cong$
- ④ reflex. prop. $\cong$
- ⑤ ASA $\cong$ (3, 4, 3)
- ⑥ CPCTC

Example:

1. Find the values of m and n.

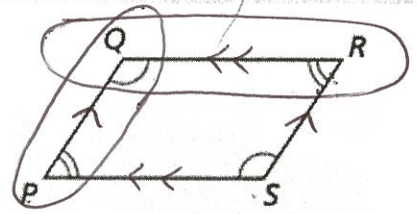


$$m = 5$$

$$n = 12$$

Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.



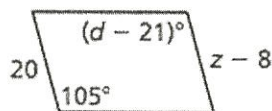
PROOF: **Given:** PQRS is a parallelogram

Prove: $\angle P \cong \angle R$ and $\angle Q \cong \angle S$

Statements	Reasons
① PQRS is a -gram	① given
② $m\angle P + m\angle Q = 180$ ($\overline{QR} \parallel \overline{PS}$) $m\angle Q + m\angle R = 180$ ($\overline{PQ} \parallel \overline{RS}$)	② if $\parallel \rightarrow$ consec. int. $\angle$ s supp.
③ $\angle P \cong \angle R$	③ $\cong$ supps thm.
④ $m\angle P + m\angle S = 180$ ($\overline{QP} \parallel \overline{RS}$) $m\angle P + m\angle Q = 180$ ($\overline{QR} \parallel \overline{PS}$)	④ if $\parallel \rightarrow$ consec. int. $\angle$ s supp
⑤ $\angle Q \cong \angle S$	⑤ $\cong$ supps thm.

Example:

2. Find the value of d



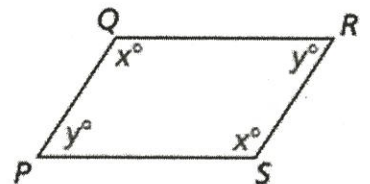
$$105 = d - 21$$

$$\boxed{d = 126}$$

Parallelogram Consecutive Angles Theorem

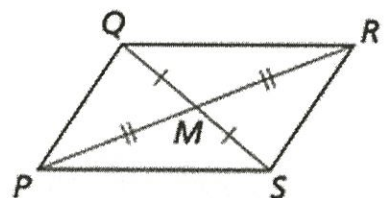
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

$$x + y = 180$$

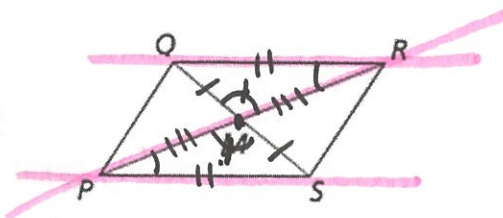


Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.



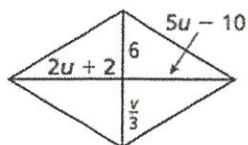
PROOF: Given: PQRS is a parallelogram
 Diagonals $\overline{PR}$ and $\overline{QS}$ intersect at M
 Prove: M bisects $\overline{PR}$ and $\overline{QS}$



Statements	Reasons
① PQRS is a -gram	① given
② $\angle QRM \cong \angle SPM$	② if two $\parallel \rightarrow$ alt. int. $\angle$ s $\cong$
③ $\angle QMR \cong \angle SMP$	③ vert. $\angle$ s $\cong$
④ $\overline{QR} \cong \overline{SP}$	④ if -gram $\rightarrow$ opp. sides $\cong$
⑤ $\triangle QRM \cong \triangle SPM$	⑤ AAS $\cong$ (2,3,4)
⑥ $\overline{PM} \cong \overline{RM}$ $\overline{QM} \cong \overline{SM}$	⑥ CPCTC
⑦ Diags intersect @ M	⑦ given
⑧ M bisects $\overline{PR}$ and $\overline{QS}$	⑧ def. of a seg. bisector.

Example:

3. Find the values of u and v.



$$2u+2 = 5u-10$$

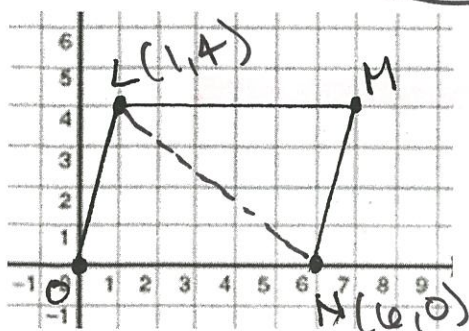
$$12 = 3u$$

$$\boxed{u = 4}$$

$$v = \frac{4}{3}$$

$$\boxed{v = 18}$$

4. Find the coordinates of the intersection of the diagonals of parallelogram LMNO.

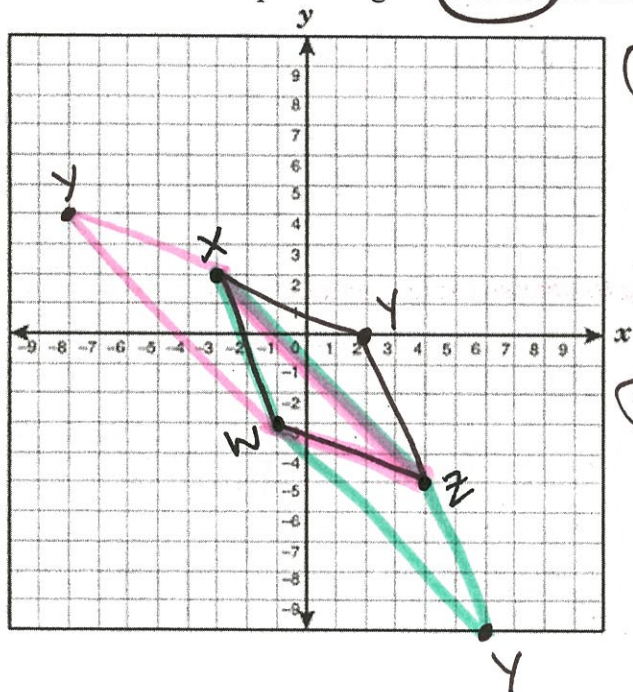


$\rightarrow$ mdpt of either diag!

$$\begin{aligned} \text{Mdpt}_{LN} &= \left(\frac{1+6}{2}, \frac{4+0}{2} \right) \\ &= \left(\frac{7}{2}, \frac{4}{2} \right) \end{aligned}$$

$$\boxed{= (3.5, 2)}$$

5. Three vertices of parallelogram $WXYZ$ are shown. Find the coordinates of the fourth vertex, Y .



① ~~order~~ order of vertices ~~matter~~ matter in name!!

*opp. sides $\parallel \rightarrow$ same slopes

$$Y(2, 0)$$

② if name is not given... consider all possible names

$$WXZY: Y(2, 0)$$

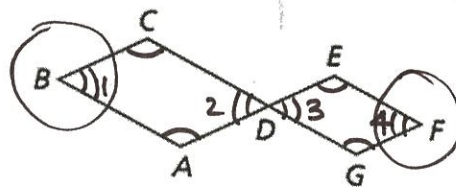
$$WZXY: Y(-8, 4)$$

$$WYZX: Y(6, -10)$$

6. **Given:** $ABCD$ and $GDEF$ are parallelograms

Prove: $\angle B \cong \angle F$

**Remember, since you know that you are given parallelograms, you can use ANY of the theorems we discussed already today! ☺ **



Statements

Reasons

① $ABCD$ and $GDEF$ are $\parallel$ -grams

② $\angle B \cong \angle CDA$
 $\angle F \cong \angle EDG$

③ $\angle CDA \cong \angle EDG$

④ $\angle B \cong \angle EDG$

⑤ $\angle F \cong \angle B$

① given

② if $\parallel$ -gram $\rightarrow$ opp. $\angle$ s $\cong$

③ vert. $\angle$ s $\cong$

④ Trans. prop. $\cong$

⑤ Trans. prop. $\cong$